Aalto University

Mat-2.4177 Seminar on case studies in operations research

Analyzing efficiency of Finnish health care units

Midterm report

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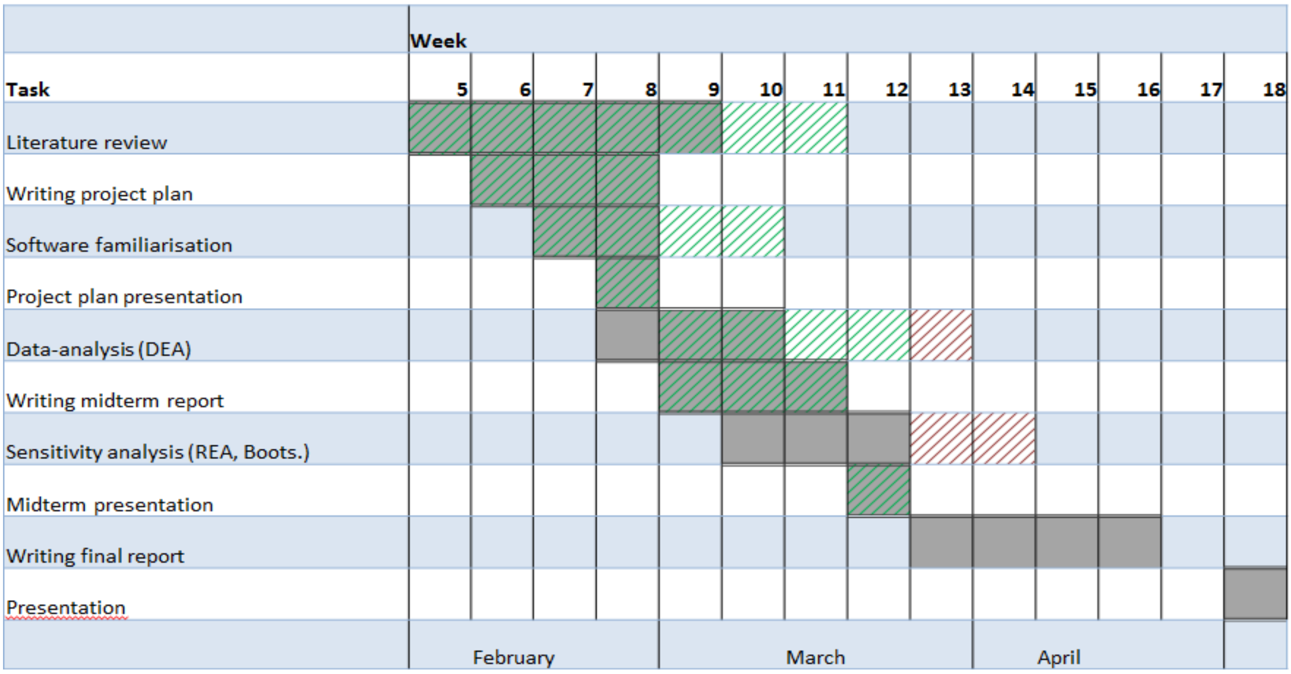
What has been done

For the midterm report, our aim was to finish introduction and literature review sections of the final report, and in addition to offer some initial results of the data analysis. The objective has been fulfilled. To familiarise ourselves with the topic, we have done extensive literature review both on the mathematical background of DEA and on efficiency studies in healthcare sector. This enabled us to gain basic knowledge about the field and to start working with the data provided by THL. Sections of the literature review include CCR, BCC (returns to scale), weight restrictions, bootstrapping, REA and previous DEA applications in the literature. We attached some example results of the researched DEA methods as appendices.

The project has advanced mostly according to the project schedule shown in table 1. Grey color indicates planned efforts and their timing as presented originally in the project plan; green indicates how they actually took place; and red indicates expected extra efforts that are still needed in order to complete a specific task. As it can be seen from the table, literature review has taken more time than anticipated, and it took longer time to familiarize ourselves with the programs available due to some compatibility problems with operating systems. Therefore, also data-analysis began one week later than planned, and we have not been able to conduct sensitivity analysis yet. On the other hand, we have made already significant progress with the final report since the literature review is already mostly in written form. To conclude, despite some minor delays, the project is progressing smoothly and it is well under control.

Table 1: Project schedule



Overview of results

A major part of this project is to make an inclusive literature review of different DEA methods. The research questions are: how various DEA methods can be applied to health care and which methods are the most suitable for different situations. After the literature review, the most prominent methods are tested with the real data provided by THL. In the next few paragraphs we explain briefly the idea of basic methods. In appendices, there are the example results of the next four methods (CCR, BCC, weight restrictions, REA).

CCR

The Charnes-Cooper-Rhodes model (CCR) (Charnes et al. 1978) is one of the most basic DEA models. The basic idea of the CCR model is that it calculates the efficiency ratio for the DMUs based on their inputs and outputs and by comparing that ratio with other DMUs defines the efficiency of the DMU.

$$\text{Efficiency ratio} = \frac{\text{virtual output}}{\text{virtual input}} = \frac{\sum_n u_n y_{nk}}{\sum_m v_m x_{mk}},$$

where y's and x's are outputs and inputs respectively and u's and v's nonnegative weights, which represent the preferences of different output and input types.

Using linear programming we determine the optimal weights, which maximize the efficiency ratio for each DMU. The optimal weights usually vary from one DMU to another. If the DMU's efficiency ratio is the best of all DMUs with some weights, the particular DMU is efficient and will have an efficiency score of one (100%). The efficient DMUs define an efficient frontier which serves as a reference in the evaluation of efficiency. (Unit B and the efficient frontier in Figure 1.)

Figure 1: A basic example of CCR model (Cooper et al. 2007)

If the DMU's efficiency ratio is not the best of all with any weights, the DMU is inefficient. The score of an inefficient DMU is always less than one and it represents how close to the efficient frontier the DMU can optimally be. The scores are always calculated with the most favorable weights for each DMU. (Cooper et al. 2007)

BCC

The previous CCR model is built on the assumption of *constant returns to scale*, meaning that if all inputs are doubled, the output is also expected to double. The Banker-Charnes-Cooper (BCC) model (Banker et al. 1984) is an extension of the CCR model and takes into account that the productivity at the most productive scale size may not be attainable for other scale sizes at which a given DMU is operating. Therefore, the BCC model estimates the pure technical efficiency of a DMU at a given scale of operation.

The only difference between the CCR and BCC models is the convexity condition of the BCC model, which means that the frontiers of the BCC model (in Figure 2) have piecewise linear and concave characteristics, which lead to *variable returns-to-scale*.

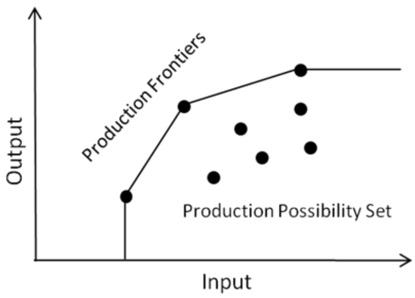


Figure 2: The BCC model

Weight restrictions

When using previous models (CCR, BCC), we might see many zeros in the optimal weights (v_i*,u_j*) of an inefficient DMU. A zero means that the particular input or output is ignored in the efficiency evaluation and it is usually a sign that the DMU has a weakness in the corresponding input/output compared with other DMUs.

There are situations where additional information is available and we want to restrict the multiplier vectors v and u more than just by non-negativity requirement. This can be done in multiple ways, for example with the assurance region approach, where we add constraints on the relative magnitudes of the weights for special items:

$$L_{1,2} \leq \frac{v_2}{v_1} \leq U_{1,2}.$$

Generally, by adding these constraints the efficiency score of a DMU is worsened and a DMU previously characterized as efficient may be found to be inefficient. Therefore one has to be careful in choosing these bounds. (Cooper et al. 2007)

REA

When using the traditional DEA methods, the efficiency scores of DMUs' represent the best possible efficiencies using the weights that are most favorable to each DMU. Traditional results do not include information how the efficiency score changes when using different input/output weights even though other weights may reflect relevant situations.

Salo and Punkka (2011) present a new method for analyzing efficiencies: the ratio-based efficiency analysis (REA). In REA method the efficiency ratios of DMUs' are evaluated with every feasible weight combination. One can then examine for example how the efficiency scores change or what the ranking intervals of the DMUs are. With this information the decision maker sees if the efficiency of a DMU is robust. REA is also suitable for situations with outliers.

Following steps

The work will be continued by executing more detailed data analysis, by iteratively developing the models and adding sensitivity analysis. Sensitivity analysis will include use of REA and possibly bootstrapping methods and analysis of data from second year for comparison. Need for further deepen the literature review will be evaluated. In discussions section several central questions of DEA applications in healthcare efficiency analysis will be addressed, such as the problem of choosing variables, for instance, qualitative vs. quantitative. Conclusions, recommendations and future research areas will be written last.

Updated risk assessment

Quickly after we began analyzing the data we encountered some difficulties in finding applicable DEA software. However, one of our references, Cooper et al. (2007) included a DEA-solver, which proved to be feasible. On the other hand, the REA-solver is provided by the Systems Analysis Laboratory as planned. That is, our software problems should be disposed of for good and the results have been promising.

In our original project plan we listed four significant risks. The time scale of the project was already discussed earlier in this report. Currently, our main concern is if our study will provide any new findings to our client. However, we have already taken action and a brief meeting with our client will be held on Friday 25th 2011. We will also discuss how to make our analysis as robust as possible. There is still a risk that we will draw premature conclusions that are not realistic.

References

Banker, R., Charnes, A., Cooper, W. 1984. Some models for estimating technical and scale inefficiencies in data envelopment analysis. *Management Science*. Vol. 30, No. 9, pp. 1078-1092.

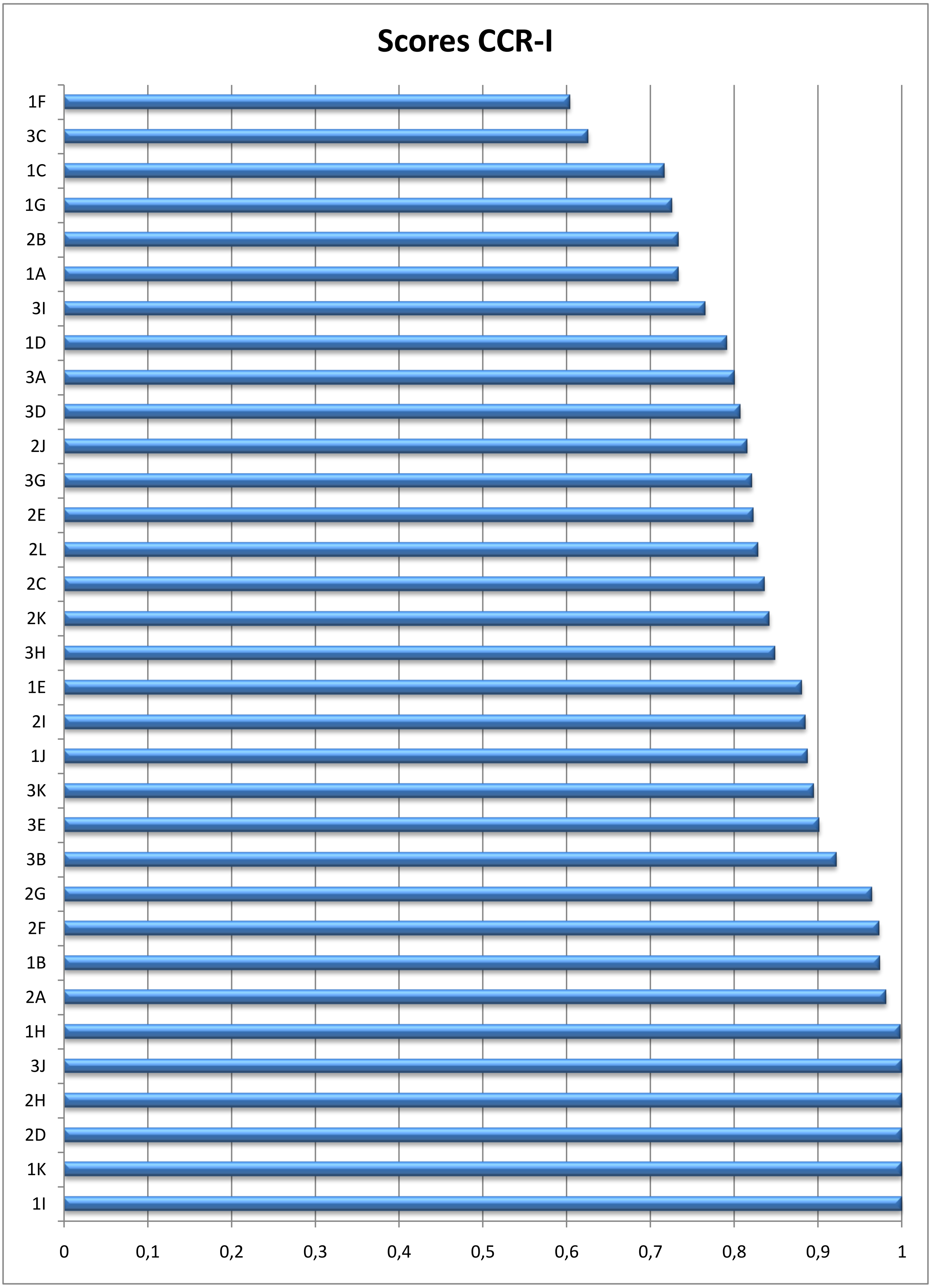
Charnes, A., Cooper, W.W., Rhodes, E. 1978. Measuring the efficiency of decision making units. *Eur. J. Oper. Res.* 2, pp. 429-444.

Cooper, W.W., Seiford, M., Tone, K. 2007. *Data Envelopment Analysis: A Comprehensive Text with Models, Applications, References and DEA-Solver Software,* 2nd ed. Springer, New York.

Salo, A., Punkka, A. 2011. Ranking Intervals and Dominance Relations for Ratio-Based Efficiency Analysis. *Management Science*, Vol. 57, No. 1, pp. 200-214.

Appendix 1: Example results, CCR-I

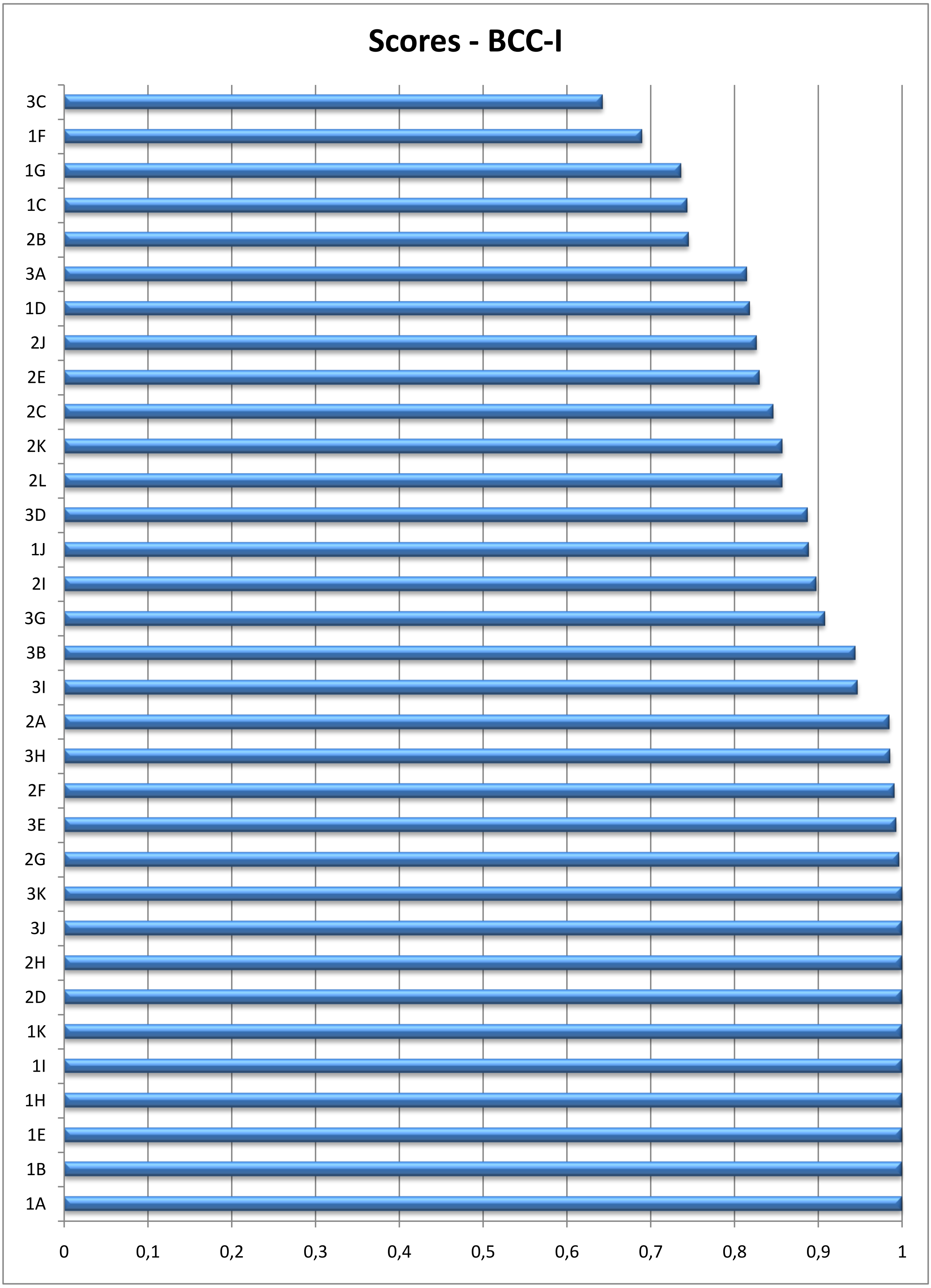
Input variables: 1) number of dentists, 2) number of dental hygienists, 3) number of dental assistants
Output variables: 1) weighted sum of completed operations (operations weighted by cost factor)
Weight restrictions: none



The units are coded: The number represents the category (1 is the biggest; categorized by the quantity of operations) and the letter separates the units in each category.

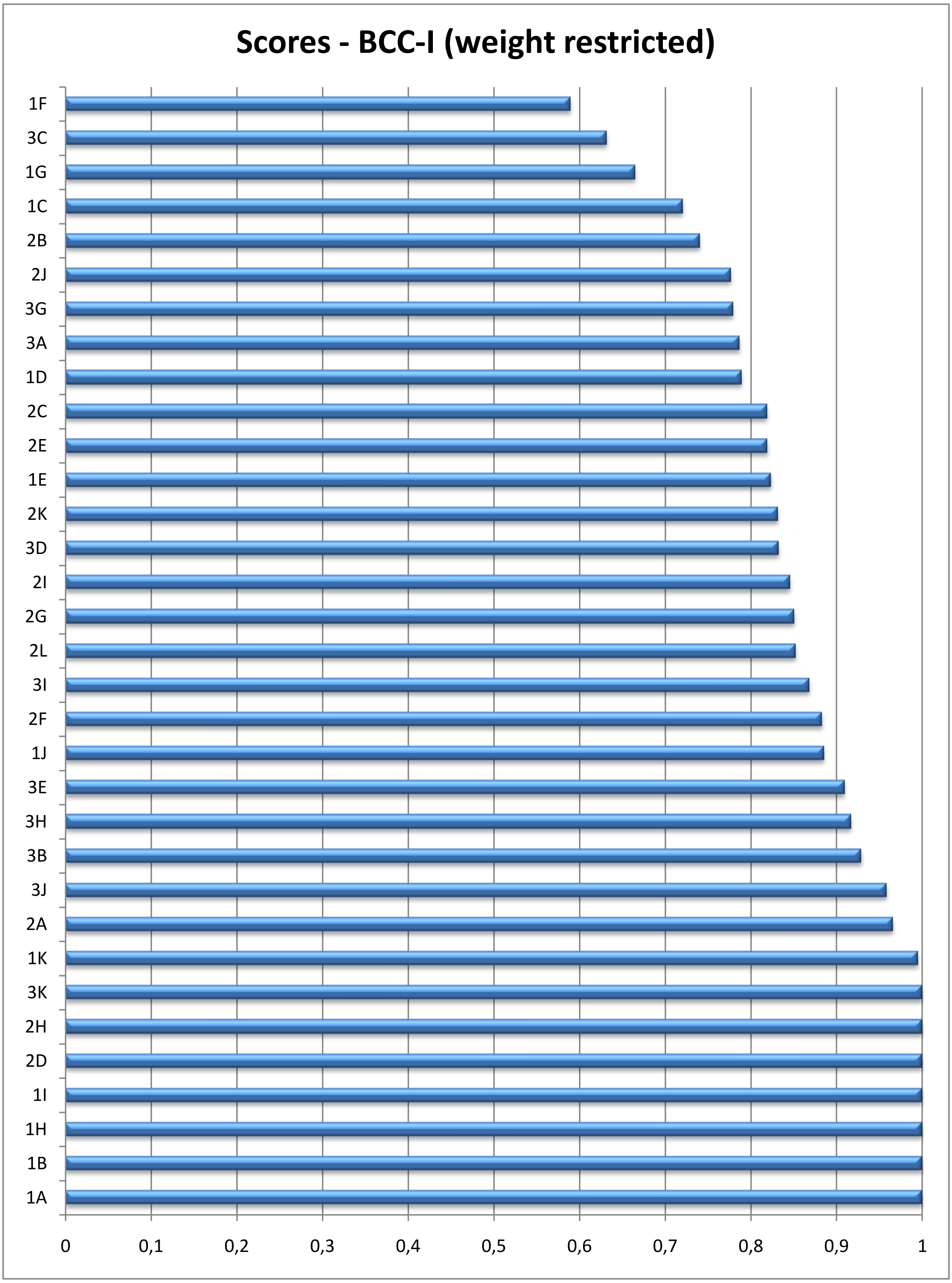
Appendix 2: Example results, BCC-I

Input variables: 1) number of dentists, 2) number of dental hygienists, 3) number of dental assistants
Output variables: 1) weighted sum of completed operations (operations weighted by cost factor)
Weight restrictions: none



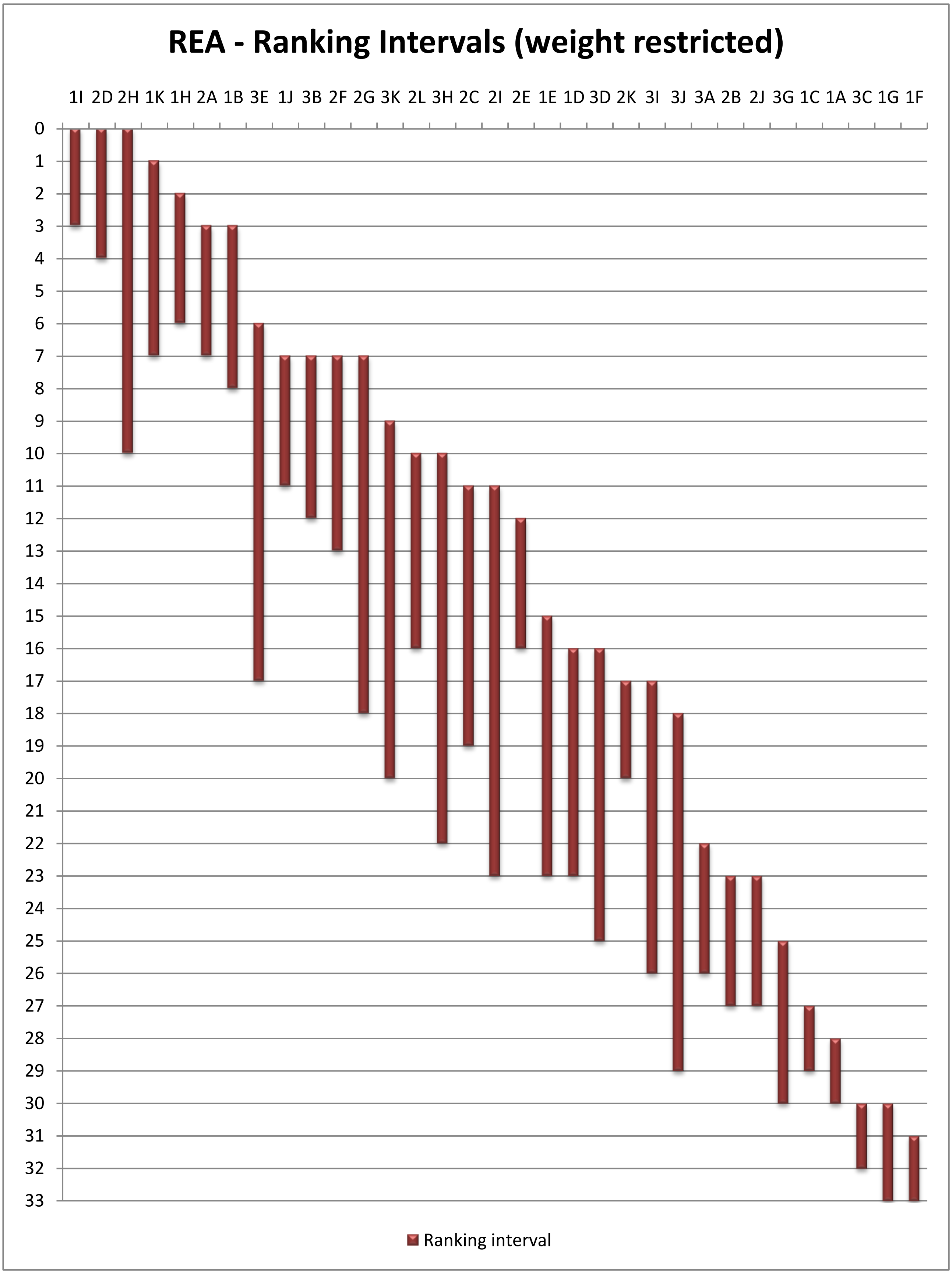
The units are coded: The number represents the category (1 is the biggest; categorized by the quantity of operations) and the letter separates the units in each category.

Appendix 3: Example results, BCC-I with weight restrictions
Input variables: 1) number of dentists, 2) number of dental hygienists, 3) number of dental assistants
Output variables: 1) weighted sum of completed operations (operations weighted by cost factor)
Weight restrictions:
1 ≤ (dentists/assistants) ≤ 5; 1 ≤ (dentists/hygienists) ≤ 5; 0,5 ≤ (hygienists/assistants) ≤ 5



The units are coded: The number represents the category (1 is the biggest; categorized by the quantity of operations) and the letter separates the units in each category.

Appendix 4: Example results, REA Ranking Intervals with weight restrictions
Input variables: 1) number of dentists, 2) number of dental hygienists, 3) number of dental assistants
Output variables: 1) weighted sum of completed operations (operations weighted by cost factor)
Weight restrictions:
1 ≤ (dentists/assistants) ≤ 5; 1 ≤ (dentists/hygienists) ≤ 5; 0,5 ≤ (hygienists/assistants) ≤ 5



The units are coded: The number represents the category (1 is the biggest; categorized by the quantity of operations) and the letter separates the units in each category.